J. Bamboo and Rattan, Vol. 4, No. 4, pp. 335–342 (2005)

Also available online - www.vsppub.com

Statistical model of strength in flexion and size effect on the failure of *Raphia vinifera* L. (Arecacea)

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Abstract—This paper deals with the statistical model of the resistance of *Raphia vinifera* L. (Arecacea) under flexion normal to the grain. After the establishment of the probability expression governing the failure of said material, we experimentally determined the parameters of the statistical law that best fits the failure. A series of tests is also carried out to determine the size effect on these parameters.

Key words: *Raphia vinifera* L. (Arecacea); probability of failure; flexion; Weibull distribution; size effect.

INTRODUCTION

Raphia vinifera L. (Arecacea) is a species of bamboo that is found predominantly in the West and North West Provinces of the Republic of Cameroon. Its extensive use in these regions as an alternative building material is due to its abundance, renewability, low cost and easy to machine finish. It also has good mechanical properties such as the young modulus, which is of the order of that of concrete [1]. Many studies have been carried out investigating the bonding between *R. vinifera* stem and concrete [2, 3]. Earlier investigations used the same statistical model on *R. vinifera* under compression, but it has been observed that it is mostly used in flexion. To the best of our knowledge little has been done on the statistical law governing the failure of *R. vinifera*. The goal of this paper is to investigate the failure in flexion. The paper is divided into four sections: establishing the probability law governing its failure, determining the relevant parameters in the statistical law, investigating the influence of sample size on these parameters and the conclusion.

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THEORETICAL BACKGROUND

R. vinifera is a ductile material [4] and the distribution function governing the failure of such materials is given by

$$F_n(\sigma) = 1 - \prod_{i=1}^{n} [1 - F(\sigma_i)]^n. \tag{1}$$

If the sample is partitioned into n identical parts; each of volume V_i, then relation (1) yields

$$F_n(\sigma) = 1 - \exp\left[-\left(\frac{1}{V_0}\right)\sum_{i=1}^{n} V_i \log(1 - F(\sigma_i))\right]. \tag{2}$$

Making use of the distribution function of an element expressed in Refs [5–7], then equation (2) becomes

$$F_n(\sigma) = 1 - \exp\left[-\left(\frac{1}{V_0}\right)\int_{V_t}\left(\frac{\sigma(x, y, z)}{\alpha}\right)^{\beta} \mathrm{d}V\right], \tag{3}$$

where V_t is that part of the volume under tension. Using the Bernoulli–Euler theory [5] and a four-point flexion loading, the normal stress distribution on the cross-section of the sample has the form:

$$\begin{cases} \sigma = \sigma_0\left(\dfrac{2xy}{Ra}\right) & \text{for } 0 \leqslant x \leqslant a \\ \sigma = \sigma_0\left(\dfrac{2y}{R}\right) & \text{for } a \leqslant x \leqslant \dfrac{L}{2}, \end{cases} \tag{4}$$

where σ_0 is the ultimate stress and R the radius of the sample. A generalized alternative of (4) is given by

$$\sigma = \sigma_0 g(x, y, z). \tag{5}$$

Putting $B_V = (1/V)\int_{V_t} g(x, y, z)^{\beta}$, the resistance distribution function yields

$$F_n(\sigma) = 1 - \exp\left(K B_V V \sigma_0^{\beta}\right), \tag{6}$$

where

$$K = \frac{1}{V_0 \alpha^{\beta}}. \tag{7}$$

The expression for B_V has been derived by Bohannan [8] for a rectangular cross-sectional area and is of the form

$$B_V = \frac{1}{2(\beta + 1)^2}\left[1 + \beta\left(\frac{L - 2a}{L}\right)\right]. \tag{8}$$

Here a and L are such that $a/L = 1/4$. For a circular cross-section, B_V is given by

$$B_V = \frac{2^{\beta-1}}{\pi(\beta+1)(\beta+2)}(3\beta+1)\int_0^{2\pi}\cos^\beta\theta\,\mathrm{d}\theta. \tag{9}$$

For any given value of β, that is, for a given material, the expression in (9) is a constant and can be evaluated using numerical methods. Thus, the mean distribution is given by [5]

$$E(\sigma) = \int_{-\infty}^{+\infty}\sigma_0\exp(KB_V V\sigma_0^\beta)x(-KB_V V\beta\sigma_0^{\beta-1})\,\mathrm{d}\sigma_0. \tag{10}$$

Putting $u = KB_V V\sigma_0^\beta$, we arrived at

$$E(\sigma) = \frac{1}{(KB_V V)^{\frac{1}{\beta}}}\Gamma\left(1+\frac{1}{\beta}\right), \tag{11}$$

where Γ is the Euler function. Similarly, the variance of the distribution takes the form

$$\mathrm{var}(\sigma) = \frac{1}{(KB_V V)^{\frac{1}{\beta}}}\left[\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)\right]^{1/2}. \tag{12}$$

Here α and β are Weibull distribution parameters that are to be determined experimentally. The sample coefficient of variation in Ref. [4] is given by:

$$\mathrm{CV} = \frac{\left[\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)\right]}{\Gamma\left(1+\frac{1}{\beta}\right)}. \tag{13}$$

This is independent of the load history. The value of β in relation (13) is determined by using the coefficient of variation. The other parameter K of the material is given in [4]:

$$k = \frac{1}{V}\left[\frac{\Gamma\left(1+\frac{1}{\beta}\right)}{\mu}\right]^\beta. \tag{14}$$

To test the quality of the adjustment we use the Chi-square test [4, 9].

EXPERIMENTAL SET-UP

The bamboo was obtained from the Dschang locality of the West Province. They were air-dried at room temperature for 35 days. In conformity with the norm B51-008 [10] and, since the *R. vinifera* was cylindrical, they were cut with a length-to-diameter ratio of 14 : 1 instead of the length-to-width ratio of 14 : 1 applied to rectangular cross-sections. 128 test pieces were randomly selected. Their surface areas and masses were determined using the compensating polar planimeter and an electronic balance, respectively. They were then oven-dried at 103°C for 3 days and their masses and moisture content determined. During the bending test, the stress was applied, incrementally, normal to the samples' grain length. Attention was focused on the ultimate stress. The instrument used was a UTM-30, which has a red indicator that indicates the value of the ultimate stress. In addition, a four-point bending with $q/L = 1/4$ was carried out to ensure uniform distribution of stress along the bar.

ANALYSIS AND RESULTS

The collected data were analysed using the normal, the two-parameter and three-parameter Weibull distributions. The latter was obtained by determining the value of the γ-parameter using the regression between strength and the oven-dry density [4]. Figure 1 shows the resulting graph and the corresponding value of γ at 12.33 MPa, while Fig. 2 is the histogram of the test results. Table 1 shows the results of the two-parameter Weibull distribution while Table 2 shows the results for the three-parameter case.

From the Chi-square evaluations, we obtained a better fit between the two-parameter Weibull distribution ($\chi^2 = 1.997$) with the observed frequencies than with either the normal ($\chi^2 = 3.4$) or the three-parameter Weibull distribution ($\chi^2 = 10.61$) [4]. This result is also shown in Fig. 3, which shows the plot of the original test data, the normal and the two-parameter Weibull distributions. These results are in conformity with those obtained by Mukam *et al.* [5, 11] for the failure

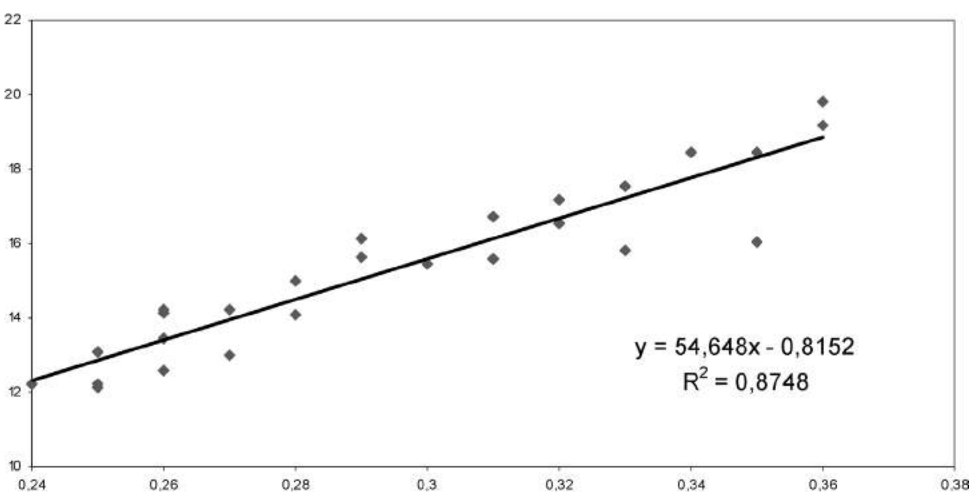

Figure 1. Bending strength in MPa (vertical axis) as a function of oven-dry density (horizontal axis).

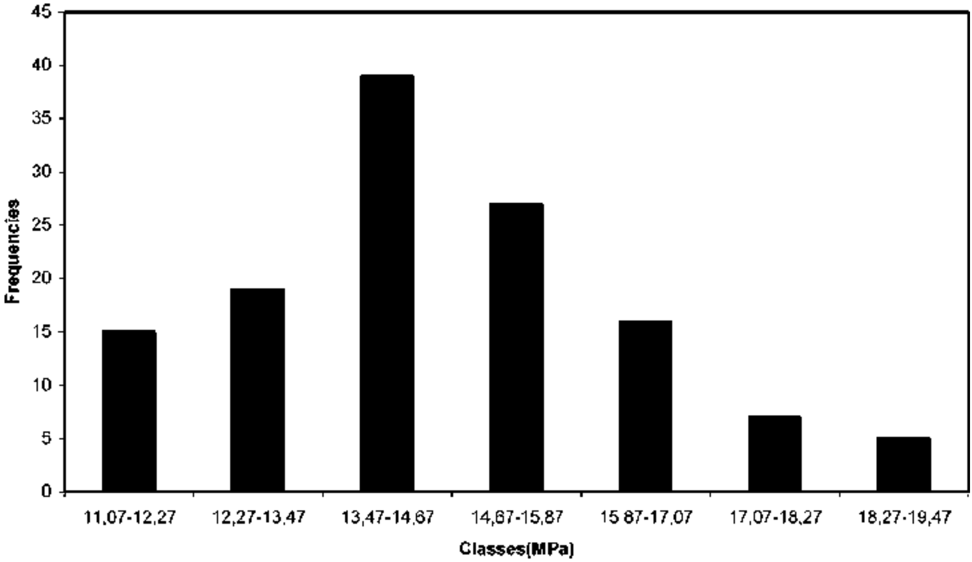

Figure 2. Frequencies distribution of the bending strength of *Raphia vinifera*.

Table 1.
Weibull distribution with two parameters

Mean of the distribution μ (MPa)	14.55
Standard deviation (MPa)	1.8
Coefficient of variation (CV, %)	12.37
α parameter	26.88
β parameter	19.71
Moisture content H (%)	18.42 ± 0.01
Number of tested pieces	128
Average volume of the test-pieces (cm^3)	236.02 ± 0.03

of wood in compression parallel to the grain and later by Talla *et al.* [4] for the failure of *R. vinifera* in compression along the grain.

The next step is analysing the effect of the size of the samples on the Weibull parameters α and β. For each radius 60 test pieces were selected. Using the procedure described earlier we determined the Weibull parameters for each group. It should be noted that we were limited to a few values of cross-sections due to the nature of *R. vinifera.* Table 3 shows the selected sections and the corresponding values of the parameters. From the table, it is observed that, the effect of the selected sections on the α parameter vary only slightly (0.7%), while that for the β parameter has a variation of about 24.6 % which is not negligible.

We can therefore conclude that the dimension of the test pieces has practically no effect on the α parameter while affecting the β parameter to an extent. Thus, care should be taken about the dimensions when modelling or evaluating the failure of building components or furniture made from split *R. vinifera*. This result is in

Table 2.
Weibull distribution with three parameters

Position parameter γ (MPa)	12.33
Mean of the distribution μ (MPa)	3.29
Standard deviation (MPa)	1.39
Coefficient of variation (CV)	45.8
α parameter	39.18
β parameter	2.32
Moisture content H (%)	18.42 ± 0.01
Number of tested pieces	94
Average volume of the test-pieces (cm^3)	236 ± 0.03
Abnormal failure	34
Abnormal failure (%)	26.6

Table 3.
Variation of statistical parameters with volume

Section (cm^2)	6.50 ± 0.2	8.50 ± 0.2	10.02 ± 0.2
Number of tested pieces	60	60	60
α parameter (MPa)	37.49	37.23	37.30
β parameter	1.93	2.10	2.56

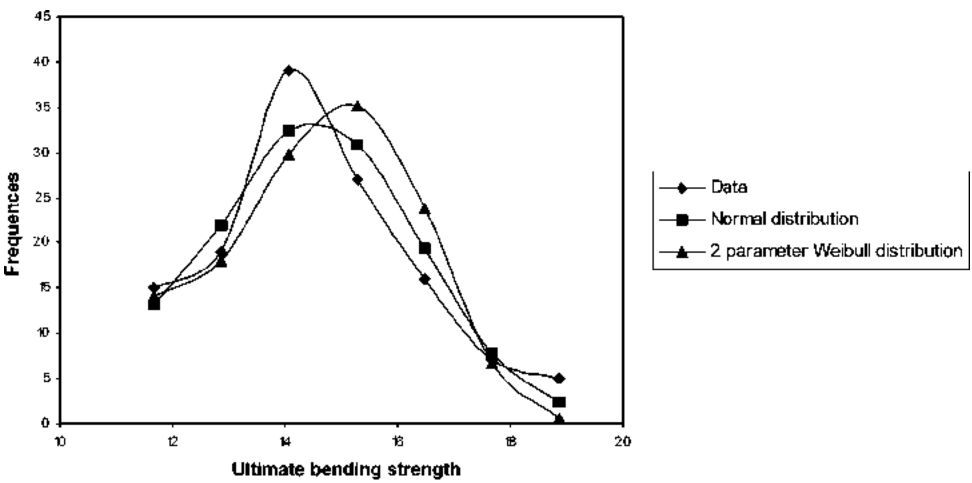

Figure 3. Comparison of the three distributions.

conformity with the graph on Fig. 4 where the cumulated frequency is plotted against resistance.

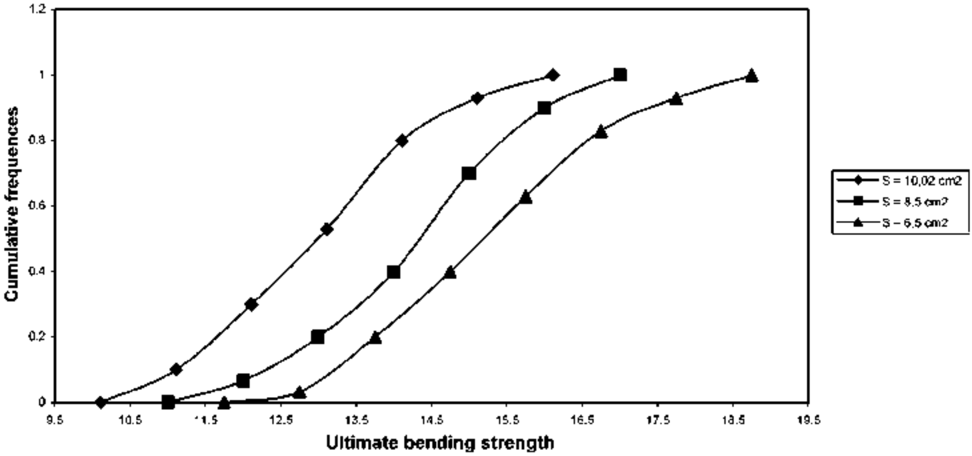

Figure 4. Cumulative distributions of the frequencies.

CONCLUSIONS

The failure of split *R. vinifera* in flexion has been investigated. In the first step we calculated the statistical coefficients related to the two-parameter and three-parameter Weibull distributions. From the chi-square calculations we arrived at the conclusion that the three-parameter Weibull distribution fits the experimental data well, and that this distribution is better suited to describe the failure of split *R. vinifera* in flexion than the two-parameter and normal distributions. The calculated statistical coefficients allow us to determine the probability of failure in flexion of test-pieces based on oven-dry density. In the second step we found that the dimensions of the split *R. vinifera* undergoing flexion have an influence on the statistic parameters. This last result suggests that particular attention should be paid on the dimensions of members of split *R. vinifera* subjected to flexion.

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